Final Exam Practice Problems

1. Consider a monopolist who has an inverse demand function given by $p(Q) = 100 - 2Q$, and a cost function $c(Q) = 2Q$. What is the firm's level of profit when they choose their profit maximizing quantity? How does the firm's output decision change if we implement a per-unit tax of τ per each unit of output produced?

2. Suppose a perfectly competitive firm is maximizing its profit in the short-run. At its profit maximizing quantity, AR > ATC. Compared to the short-run, in the long- run there will be _____ firms in the market, and each firm will produce _____ quantity

3. Consider a perfectly competitive market with N identical firms, each having a cost function $c(q) = 50 + 2q^2$. The market demand curve is given by $p(Q) = 100 - 2Q$. Determine the long run equilibrium (p^*, q^*, N^*) in this market, where (p^*, q^*, N^*) represent the equilibrium price, the equilibrium quantity produced by each firm, and the total number of firms in the market.

4. You've solve cost minimization problems for firms, now you'll do it for a consumer. A consumer has a utility function given by $u(x, y) = xy$. Suppose $p_x = p_y = 1$. Solve the consumer's expenditure minimization problem such that their utility is at least $\bar{u} = 25$. How much does the consumer spend?

1 Solutions

1. The firm's profit function is

$$\pi(Q) = (100 - 2Q)Q - 2Q$$

To solve, we take the first order condition with respect to Q, set equal to 0, and solve for Q. The FOC is

$$100 - 4Q - 2 = 0$$

Solving for Q yields

$$Q^* = \frac{98}{4} = 24.5$$

To find the firm's profit, simply plug this into the profit function:

$$\pi(Q^*) = (100 - 2(\frac{98}{4}))\frac{98}{4} - 2(\frac{98}{4}) = 2499 - 49 = 2450$$

Now, suppose the firm faces a per unit tax of τ per unit of output. The profit function is now

$$\pi(Q) = (100 - 2Q)Q - 2Q - \tau Q$$

and the solution is

$$Q^* = \frac{98 - \tau}{4} < \frac{98}{4}$$

so the firm reduces output.

2. A competitive firm's profit function is

$$\pi(q) = pq - c(q)$$

where $c(q)$ is just a general cost function. The profit maximizing condition is where marginal revenue (MR) equals marginal cost (MC).

$$p = c'(q)$$

We are told $AR > ATC$. Now

$$AR = \frac{pq}{q} = p = MR$$

$$ATC = \frac{c(q)}{q}$$

So we have

$$p > \frac{c(q)}{q}$$

$$pq > c(q)$$

$$TR > TC$$

$$\pi(q) > 0$$

Since there are positive profits, more firms will enter the market, depressing the price, and reducing each firm's individual output.

3. The conditions for a market equilibrium are as follows:

$$p = c'(q)$$

$$p = \frac{c(q)}{q}$$

$$Q^S = Q^D$$

We start by solving each individual firm's problem. The profit function for firm i is

$$\pi(q) = pq - (50 + 2q^2)$$

The first order condition is

$$p - 4q = 0$$

or

$$p = 4q$$

Now, we also know

$$p = \frac{c(q)}{q}$$

$$p = \frac{50}{q} + 2q$$

Set the terms containing p equal to one another:

$$4q = \frac{50}{q} + 2q$$

$$2q = \frac{50}{q}$$

$$2q^2 = 50$$

$$q^2 = 25$$

$$q^* = 5$$

Now $p = 4q^*$ so

$$p^* = 20$$

We can plug this into the demand curve:

$$p^* = 100 - 2Q^D$$

$$20 = 100 - 2Q^D$$

$$2Q^D = 80$$

$$Q^D = 40$$

Market supply is given by

$$\sum_{i=1}^{N} q_i^* = Nq_i^*$$

and market demand is 40.

$$Nq_i^* = 40$$
$$5N = 40$$
$$N^* = 8$$

The solution is then the tuple $(p^*, q^*, N^*) = (20, 5, 8)$

4. The consumer's total expenditure is just how much of x and y they buy times the cost

$$e(x, y) = p_x x + p_y y$$

We want to minimize $e(x, y)$ such that

$$xy \geq 25$$

The Lagrangian is

$$\ell = p_x x + p_y y - \lambda(xy - 25)$$

The first order conditions yield

$$p_x - \lambda y = 0$$
$$p_y - \lambda x = 0$$
$$xy = 25$$

Isolating λ yields and equating

$$\frac{p_x}{y} = \frac{p_y}{x}$$

We know $p_x = p_y = 1$

$$\frac{1}{y} = \frac{1}{x}$$

So the expenditure minimizing quantities occur when

$$x = y$$

We replace this into our constraint

$$xy = 25$$
$$x^2 = 25$$
$$x^* = y^* = 5$$

So the total cost to the consumer is 10.