Marginal Rate of Substitution

1 Computing the MRS at a Bundle

We know that if we have a utility function $u(x, y)$, that the marginal rate of substitution is given by

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{MU_x}{MU_y}$$

The MRS tells us how many units of good y we must give up to increase our consumption of good x by 1 unit and keep utility unchanged. An example of a question could be: A consumer has preferences over goods x and y that are represented by the utility function

$$u(x, y) = x^{\frac{1}{2}} y^{\frac{2}{3}}$$

According to the MRS, what is the maximum amount of good y this consumer is willing to give up to obtain one more unit of x, if their current bundle is (4,8)?
How do we answer this question? First, is find the MRS. We derive

$$MU_x = \frac{1}{2} x^{-\frac{1}{2}} y^{\frac{2}{3}}$$

$$MU_y = \frac{2}{3} x^{\frac{1}{2}} y^{-\frac{1}{3}}$$

so the MRS is given by

$$MRS = \frac{MU_x}{MU_y} = \frac{3y}{4x}$$

Now, we are able to identify the MRS for any points (x,y). To answer how much of good y we are willing to give up to increase our consumption of good x by 1 unit and be just as well off, we plug in these values:

$$MRS_{(4,8)} = \frac{3(8)}{4(4)} = \frac{3}{2}$$

This means, at this point, the consumer is willing to give up $\frac{3}{2}$ units of y to obtain an additional unit of x. You could also say they are willing to give up 3 units of y to obtain 2 units of x. What would the answer be if the current consumption bundle was (6,3)?

2 Comparative Statics

We can also analyze the properties of the MRS by moving along the indifference curve, as the slope of the indifference curve is the negative of the MRS. Suppose we are given a utility function

$$u(x, y) = xy$$

the MRS is then

$$MRS = \frac{y}{x}$$

How does the MRS change as we vary x and y? We can answer this by taking partial derivatives.

$$\frac{\partial MRS}{\partial x} = -\frac{y}{x^2} < 0$$

this means that the MRS is diminishing in good x. What does this mean for our consumer though? Suppose we are initially at the bundle (2,2), the MRS is simply 1. Now, we increase x to 3, so our new bundle is (3,2), the MRS at this bundle is $\frac{2}{3}$ which is less than 1. At (2,2), the consumer is willing to give up 1 unit of y to obtain an additional unit of x, however, at (3,2), they are only willing to give up $\frac{2}{3}$ units of y to obtain an additional unit of x. This is what it means for the MRS to be diminishing in good x, as we increase our consumption of x, we are willing to give up less y to obtain an additional unit.