Uncertainty Problems

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1 Discrete Choice Problems

Discrete choice problems are those where our agents can either decide to take an action or not to take an action.

1. Suppose an agent is deciding whether or not to go to college. If they go to college, there is a 0.7 probability that their wage is 2 and a 0.3 probability that their wage is 1. If they do not attend college, their wage is 1. The cost of attending college is 0.6. Should this individual attend college? To anwer this, we must see if the expected benefit (here the expected wage) exceeds the cost of attending. The agent will attend if

   $$E[w|college] - c > 1$$

   or

   $$0.7(2) + 0.3(1) - 0.6 > 1$$

   Since the expected wage from attending college in this case is 1.7 and the cost is 0.6, this individual will attend college.

2. Another example problem I could give you here would be: to find the condition on the cost c such that this individual would attend college. In this case, we are solving

   $$E[w|\text{college}] - c > E[w|\text{no college}]$$

   for some condition on c. In this case, we would have

   $$0.7(2) + 0.3(1) - c > 1$$

   or

   $$1.7 - 1 > c$$

   $$0.7 > c$$

   So as long as the cost is less than 0.7, the agent will go to college.

2 Endogenous Probability

1. Suppose we have a self employed individual who chooses an effort level e to maximize their expected utility. There are two possible realizations of output V_h and V_l with $V_h > V_l$. The probability of the high outcome is a fucntion of their effort

   $$p(V = V^h) = \frac{e}{1+e}$$

   and is increasing as a function of effort. Suppose the cost of effort is linear, therefore

   $$c(e) = e$$

What is the solution to the agent's problem? We start by forumlating the maximization problem the consumer will choose e to maximize the following:

$$V^h \frac{e}{1+e} + V^l \frac{1}{1+e} - e$$

Taking the derivative with respect to e and setting equal to 0 yields

$$V^h \frac{(1+e)-e}{(1+e)^2} + V^l \frac{-1}{(1+e)^2} - 1 = 0$$

or

$$V^h \frac{1}{(1+e)^2} + V^l \frac{-1}{(1+e)^2} - 1 = 0$$

$$V^h \frac{1}{(1+e)^2} + V^l \frac{-1}{(1+e)^2} = 1$$

$$V^h - V^l = (1+e)^2$$

$$\sqrt{V^h - V^l} = (1+e)$$

$$\sqrt{V^h - V^l} - 1 = e^*$$

Therefore, for this agent to exert a positive level of effort it must be that $\sqrt{V^h - V^l} > 1$

3 Insurance Questions

1. A consumer is deciding how much insurance to purchase. Assume the probability of an accident is θ and if the accident occurs they incur a loss of L. The premium is p and the amount chosen is c. Assume that our consumer starts with an additional level of wealth, w. Their utility function is given by

$$u(x) = ln(x)$$

We want to solve for the choice of coverage, c as a function of other model parameters, p, w, and L. The consumer's maximization problem is to choose c to maximize.

$$\theta u(\text{accident}) + (1-\theta)u(\text{no accident})$$

$$\theta u(w - L + c - pc) + (1-\theta)u(w - pc)$$

So how did we get the terms inside the utility functions? Our consumer initially has wealth w. If they purchase c units of coverage at premium p, and no accident occurs, their new level of wealth is w -pc. If the accident occurs, they additionally incur the loss L, but they are given c (the coverage they purchased to insure against the loss). Taking first order conditions with respect to c and setting equal to 0 yields

$$\theta \frac{1-p}{w - L + c - pc} + (1-\theta)\frac{-p}{w - pc} = 0$$

$$\theta \frac{1-p}{w - L + c - pc} = (1-\theta)\frac{p}{w - pc}$$

$$(w - pc)\theta(1-p) = (1-\theta)p(w - L + c - pc)$$

$$w\theta(1-p) - pc(1-p) = (1-\theta)p(w - L) + c(1-p)(1-\theta)p$$

$$cp(1-p) = w\theta(1-p) - (w - L)p(1-\theta)$$

$$c^* = \frac{w(\theta(1-p) - p(1-\theta)) + Lp(1-\theta)}{p(1-p)}$$

Note that coverage c is increasing in the amount of the loss, L, which makes sense. The more we stand to lose, the more coverage we buy.

4 Risk Aversion

Recall that the coefficient of relative risk aversion is given by

$$A(x) = -\frac{u''(x)}{u'(x)}$$

1. Let u(w) = $\frac{w^{1-\alpha}}{1-\alpha}$ Show that u(w) has a constant coefficient of risk aversion equal to α To solve this, we directly use the formula for A(x)

$$u'(w) = (1-\alpha)\frac{w^{-\alpha}}{1-\alpha} = w^{-\alpha}$$
$$u''(w) = -\alpha w^{-\alpha-1}$$
$$A(x) = -\frac{-\alpha w^{-\alpha-1}}{w^{-\alpha}}$$
$$A(x) = \frac{\alpha}{w}$$

So what happens to this consumers risk aversion as their wealth increases?

$$\frac{dA(w)}{dw} = -\frac{\alpha}{w^2} < 0$$

so risk aversion decreases the wealthier they are.

2. How do you show an agent is risk averse? We check the second derivative of u(w). So in the previous example

$$u''(w) = -\alpha w^{-\alpha-1} < 0$$

so the agent is risk averse.

5 Risky Assets

1. We can also derive how much we would be willing to invest in a risky asset. Suppose our initial level of wealth is w and we have the opportunity to invest x into some risky asset. With probability p, we earn a rate of return of r. With probability 1-p we get nothing, how much should we invest if our utility function is $u(w) = \sqrt{x}$?

In this problem, the choice variable is x, the amount to invest. If we invest, and we earn a positive return, our new wealth is

$$w - x + x(1+r)$$

as we invested x but now get x(1+r) in return (initial investment plus return). If the investment does not yield a return, our wealth is now w-x. The maximization problem we solve is

$$\max_x p\sqrt{w+xr} + (1-p)\sqrt{w-x}$$

Taking first order condition and setting equal to 0 yields

$$\frac{rp}{2}(w+xr)^{-\frac{1}{2}} + \frac{1-p}{2}(w-x)^{-\frac{1}{2}}(-1) = 0$$
$$\frac{rp}{\sqrt{w+xr}} = \frac{1-p}{\sqrt{w-x}}$$
$$rp\sqrt{w-x} = (1-p)\sqrt{w+xr}$$
$$(rp)^2(w-x) = (1-p)^2(w+xr)$$
$$x(rp)^2 + xr(1-p)^2 = w(rp)^2 - w(1-p)^2$$
$$x^* = \frac{w((rp)^2 - (1-p)^2)}{(rp)^2 + r(1-p)^2}$$