Final Exam

July 29, 2024

For full credit, please show all work.

1 Question 1

A consumer's utility function is given by $u(x, y) = xy$. This consumer faces a generic budget constraint

$$p_x x + p_y y = I$$

(a) Determine the demand functions $x^*(p_x, p_y, I)$ and $y^*(p_x, p_y, I)$.

(b) Determine this consumer's indirect utility function $v(p_x, p_y, I)$

2 Question 2

Consider a market with two firms called Firm 1 and Firm 2. The inverse demand curve is given by $p(Q) = a - q_1 - q_2$. The two firms do NOT have identical cost functions. Suppose each firms cost function is $c_i(q_i) = c_i q_i$. Assume that $a > c_1$ and $a > c_2$. Finally, assume $c_1 > c_2$.

(a) Solve for the Cournot equilibrium levels of output (q_1^*, q_2^*). Which firm produces more? Why?

(b) Now solve for the Stackelberg equilbrium levels of output (q_1^*, q_2^*) where Firm 1 moves first. Does Firm 1 produce more or less in the Stackelberg Game than they do in the Cournot Game? Meaning, does the first mover advantage still exist in this problem even though costs are asymmetric?

3 Question 3

Determine all Nash Equilibria for the following games, if there are any.

(a)

Player 2

		A	B
Player 1	A	$10, 10$	$0, 0$
	B	$0, 0$	$10, 10$

(b)

Player 2

		A	B	C
	A	$1, -2$	$0, 0$	$3, 2$
Player 1	B	$2, -1$	$-1, -1$	$2, 3$

(c)

Player 2

		A	B
Player 1	A	$1, -1$	$-1, 1$
	B	$-1, 1$	$1, -1$

4 Question 4

Consider a perfectly competitive market with N identical firms, each having a cost function $c(q) = 50 + 2q^2$. The market demand curve is given by $p(Q) = 100 - 2Q$. Determine the long run equilibrium (p^*, q^*, N^*) in this market, where (p^*, q^*, N^*) represent the equilibrium price, the equilibrium quantity produced by each firm, and the total number of firms in the market.

5 Question 5

Consider a monopolist who has a cost function $c(q) = cq$. With probability 0.5, they think the inverse demand curve is given by p(q) = a - bq, and with probability 0.5 they think the inverse demand curve is p(q) = a - rq. Assume $a > c$ and $b > r$.

(a) Set up the monopolist's expected profit maximization problem.

(b) Solve for the expected profit maximizing quantity q^*.

6 Question 6

Consider an agent who has a single unit of time and must divide their time between leisure ℓ and work h. The agent likes leisure and enjoys consumption, c. For each unit they work, they recieve a wage, w. Their utility function is

$$u(\ell, c) = ln(\ell) + ln(c)$$

The constraints they face are

$$c = wh$$

$$1 = \ell + h$$

This implies, you can re-write their utility function as

$$u(\ell, h) = ln(\ell) + ln(wh)$$

(a) Set up this agent's Lagrangian. The choice variables are ℓ and h.

(b) Prove that the optimal choice for this agent is $(\ell^*, h^*) = (\frac{1}{2}, \frac{1}{2})$